

SMITH PREDICTOR FOR SOPDT SISO PROCESSES WITH RESONANT POLES

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Abstract

In this paper, the design and tuning controller for SOPDT SISO processes with resonant poles was presented. Also, a simple procedure for automatic tuning parameters of proposed controller was defined.

Keywords: SOPDT, Smith predictor, resonant poles, automatic tuning.

INTRODUCTION

Industrial processes with long dead-time cannot be effectively controlled with simple PID controller. The main reason for this that the additional phase lag caused by dead-time tends to destabilize closed-loop control system, reducing the gain and phase margin. Therefore, parameters of PID controller have to be limited to preserve stability, which effectively reduces loop performance. Smith predictor [1] is most widely used controller for compensating effects of large dead-time. A good property of the Smith predictor is the elimination of the dead-time from the characteristic equation of the closed-loop system. Generally, major drawbacks of Smith predictor is large number of controller parameters (because that the controller explicit contain model of the process) and sensitivity to modelling errors.

In basic form, presented in Fig. 1, Smith predictor is applicable for stable processes. In [2] authors presented so called PIP controller (PI Predictive) for SISO processes. It is PI controller with FOPDT (First Order Plus Dead Time) model. PIP controller has small number of adjustable parameters with clear physical interpretation, which can be easy tuned manually. This is very important feature for industrial application. In [3] a simple and effective procedure for automatic tuning of PIP controller was defined.

In case of integrative or unstable processes, for efficiently eliminating effect of load disturbances, some modifications of original Smith predictor structure must be made. Some solutions of this problem have great possibilities for industry application, because of simplicity, robustness and small number of adjustable parameters which have clear physical interpretation [4], [5].

It should be noted here that in past decade Smith predictor and it's modifications are attracted much attention of researchers all around the world for industry application [6].

In this paper we consider design and tuning a Smith predictor for SISO (Single Input Single Output) SOPDT (Second Order Plus Dead Time) processes with resonant poles. Also in this paper a simple and effective procedure for automatic tuning of proposed controller was defined.

EXPOSITION

The control system structure of the Smith predictor for SISO SOPDT process with resonant poles was presented in Fig.1, where

$$G_p(s) = \frac{K_p \omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2} e^{-t_k}, \quad (1)$$

$$G_{m1}(s) = \frac{K_{mp} \omega_{mn}^2}{s^2 + 2\zeta \omega_{mn} s + \omega_{mn}^2}, \quad (2)$$

$$G_{m2}(s) = e^{-\tau}, \quad (3)$$

$$G_r(s) = K_r \left(1 + \frac{1}{T_i s} + T_d s \right). \quad (4)$$

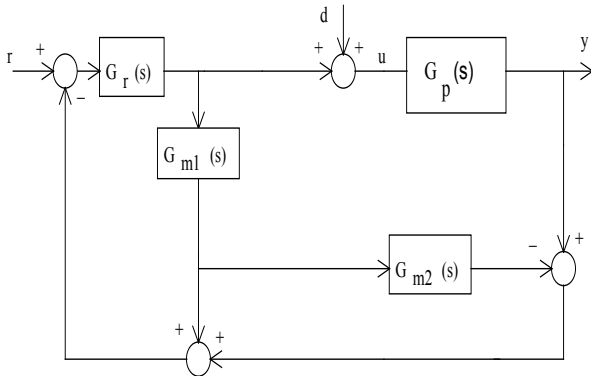


Fig. 1. Basic form of Smith predictor

As in [2], for design a controller we assume a case of perfect modeling, i.e.

$$K_p = K_{mp}, \omega_n = \omega_{mn}, \zeta = \zeta_m, \theta = \tau. \quad (5)$$

The setpoint and load disturbance response is given by

$$Y(s) = H_r(s)R(s) + H_d(s)D(s). \quad (6)$$

Design goal is that the transfer function $H_r(s)$, which defines setpoint response, will be FOPDT (First Order Plus Dead Time). Using the pole-zero cancellation design method, i.e. if we assume that following relation holds true

$$\frac{1}{\omega_{mn}^2} s^2 + 2 \frac{\xi_m}{\omega_{mn}} s + 1 = T_i T_d s^2 + T_i s + 1, \quad (7)$$

then the transfer function $H_r(s)$ is given by

$$H_r(s) = \frac{K_{mp} K_r e^{-\tau s}}{T_i s + K_{mp} K_r}. \quad (8)$$

With condition given in (7), the transfer function $H_d(s)$, which defines load disturbance response, is given by

$$H_d(s) = \frac{K_{mp} \omega_{mn}^2 e^{-\tau s}}{s^2 + 2 \xi_m \omega_{mn} s + \omega_{mn}^2} \frac{T_i s + K_{mp} K_r (1 - e^{-\tau s})}{T_i s + K_{mp} K_r}. \quad (9)$$

From relation (8) and (9) follows that

$$\lim_{s \rightarrow 0} H_r(s) = 1, \quad (10)$$

$$\lim_{s \rightarrow 0} H_d(s) = 0, \quad (11)$$

Relation (10) and (11) shows that, if the closed loop system defined by (6), (8) and (9) is

asymptotically stable, then control system follows reference and eliminate disturbance without steady-state error.

Relation (7) defines tuning formulas for parameters T_i and T_d of PID controller

$$T_i = 2 \frac{\xi_m}{\omega_{mn}}, \quad (12)$$

$$T_d = \frac{1}{T_i \omega_{mn}^2} = \frac{1}{2 \xi_m \omega_{mn}}. \quad (13)$$

Setpoint response, according to (8), is defined by real pole

$$s = -\frac{K_{mp} K_r}{T_i} = -\frac{K_{mp} \omega_{mn}}{2 \xi_m} K_r \quad (14)$$

Load disturbance response, according to (9), defined by real pole (14) and resonant poles defined by

$$s_{2,3} = -\xi_m \omega_{mn} \pm j \omega_{mn} \sqrt{1 - \xi_m^2}. \quad (15)$$

As we can see from (15) location of resonant poles is fixed by model parameters ξ_m and ω_{mn} . From (14), it follows that location of real pole is defined by three fixed model parameters K_{mp} , ξ_m and ω_{mn} and one tunable parameter, gain of PID controller K_r . Thus, varying the K_r will move the real pole (14) along the negative part of real axis in s-plane. If we chose location of real pole (14) so that to be equal the real part of resonant poles defined in (15)

$$s_1 = -\xi_m \omega_{mn} \quad (16)$$

then it follows from (14) and (16) tuning formula for parameter K_r of PID controller

$$K_r = \frac{2 \xi_m^2}{K_{mp}} \quad (17)$$

Relations (12), (13) and (17) along with relations (2) and (3) define Smith predictor for SOPDT SISO processes with resonant poles. Proposed controller is completely defined with four parameters K_r , ω_{mn} , ξ_m and τ . Resonant poles in (9) are fixed by proposed design procedure, but real pole in (8) and (9) defined in (14) are adjustable, depend on its location on negative part of real axis in s-

plane. In our design we choose the pole location defined in (16).

Discretization of proposed controller for digital implementation is straightforward. If we assume that the D/A converter is zero-order hold circuit and the sampling period is Δt , then digital Smith predictor was defined by following relations [7]:

$$G_{m1}(z) = Z\{G_{ho}(s)G_{m1}(s)\} \quad (18)$$

$$G_{m2}(z) = z^{-N}, \quad N = \frac{\tau}{\Delta t} \quad (19)$$

$$G_r(z) = \frac{u_r(z)}{e(z)} \quad (20)$$

$$u_r(z) = u_p(z) + u_i(z) + u_d(z) \quad (21)$$

$$u_p(z) = K_r e(z) \quad (22)$$

$$u_i(z) = \frac{K_r \Delta t}{T_i} \frac{z^{-1}}{(1 - z^{-1})} e(z) \quad (23)$$

$$u_d(z) = \frac{K_r T_d}{\Delta t} (1 - z^{-1}) e(z) \quad (24)$$

According to widely used rules defined in [7], we adopt small sampling period Δt .

In this place, we may define a simple and effective procedure for automatic tuning parameters of proposed controller. Procedure consists a experiment with step response of SOPDT process. This is presented in Fig. 2. If we adopt adequate percent D_p then we can determine the time when response $y(t)$ reaches the value $D_p y_s$, where the y_s denote a steadystate value of response $y(t)$, i.e. it's discrete time equivalent $y(k\Delta t)$, $k = 0, 1, 2, 3, \dots$. So determined time define parameter τ of controller. This is presented in Fig. 2. where

$$y(k_d \Delta t) = D_p y_s, \quad \tau = k_d \Delta t \quad (25)$$

From Fig. 2 we can easily define parameter K_{mp} of controller as

$$K_{mp} = \frac{y_s}{A} \quad (26)$$

where A denote amplitude of input step signal for experiment. Also, from step response presented in Fig. 2 we can easily determine value of overshoot Π and time when overshoot occurs, i.e. time interval t_1 .

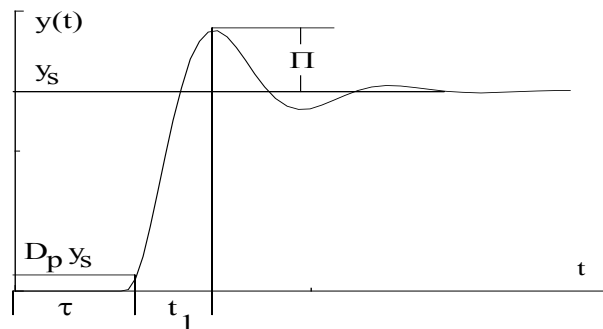


Fig. 2. Experiment for determining parameters of Smith predictor.

Then, using well known formulas we obtain two parameters ξ_m and ω_{mn} , which defines the model $G_{m1}(s)$, relation (2), of Smith predictor and parameters of PID controller $G_r(s)$, relation (4),

$$\xi_m = \frac{\ln\left(\frac{1}{\Pi}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{1}{\Pi}\right)}}, \quad (27)$$

$$\omega_{mn} = \frac{\pi}{t_1 \sqrt{1 - \xi_m^2}}. \quad (28)$$

In case when a measurement noise is present, we first filter the step response signal $y(t)$. For example, we may use centered MA (Moving Average) filter.

For performance analysis of Smith predictor and testing the procedure for automatic tuning, we use digital variant of proposed controller. Two processes will be considered, which can be described by following transfer functions

$$G_{p1}(s) = \frac{3}{s^2 + s + 3} e^{-5s}, \quad (29)$$

$$G_{p2}(s) = \frac{3(-s+1)}{s^2 + s + 3}. \quad (30)$$

Process $G_{p1}(s)$ illustrate assumptions of perfect modelling, while process $G_{p2}(s)$ illustrate significant effects of unmodelled dynamics. For both processes, procedure for automatic tuning of proposed controller was applied. Variance of measurement noise is $\sigma^2 = 4 \cdot 10^{-3}$. Sampling period is $\Delta t = 0.1$. A unit step setpoint is introduced at time $t = 0$ and a constant load disturbance $d = -0.3$ is introduced at time $t = 40$. Parameter D_p , relation (25), is $D_p = 0.05$ (5%).

Fig. 3. illustrate performance of control system for process $G_{p1}(s)$. Parameters of controller was estimated from MA (Moving Average) filtered step response are $\xi_m = 0.332$, $\omega_{mn} = 1.666$, $\tau = 5$, $K_{mp} = 1.06$.

Fig. 4. illustrate performance of control system for process $G_{p2}(s)$. Parameters of controller was estimated from MA filtered step response are $\xi_m = 0.178$, $\omega_{mn} = 2.281$, $\tau = 1.2$, $K_{mp} = 1$.

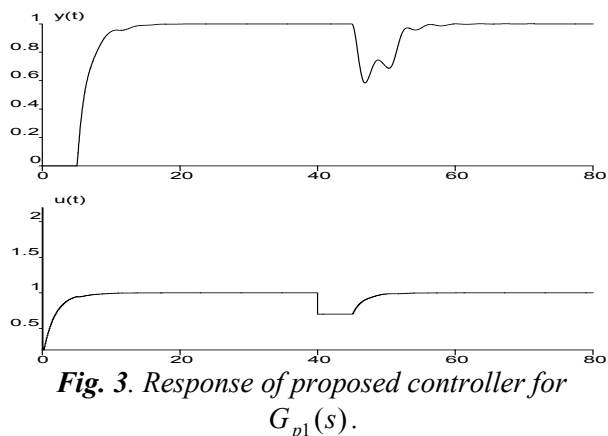


Fig. 3. Response of proposed controller for $G_{p1}(s)$.

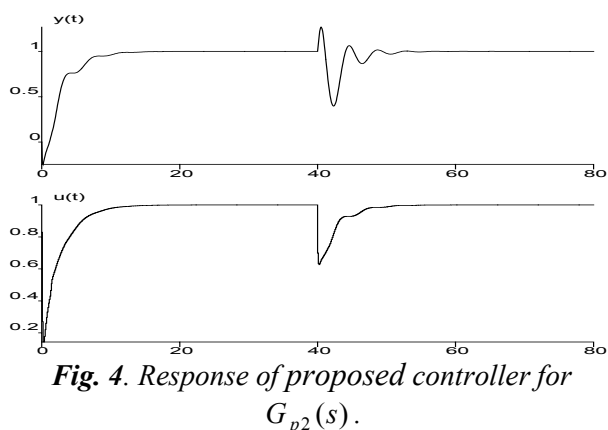


Fig. 4. Response of proposed controller for $G_{p2}(s)$.

CONCLUSION

A Smith predictor for controlling a process with resonant poles and long dead-time is proposed in this paper. The controller structure is quiet simple. Proposed tuning formulas enable to apply a simple procedure for automatic tuning of control system. Simulations performed have shown that high closed-loop system performances can be obtained.

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