

ANALYSIS OF THE INTERACTION OF TWO SOLITONS AND VIOLATION OF THEIR LINEAR SUPERPOSITION IN THE TRANSMISSION PATH

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Abstract

In this paper presented analysis of interaction of two solitons in the transmission path, and based on the soliton solution Korteweg-de Vries equation. Shows the simulation of interaction of two solitons when they are in a certain mutual distance. Also shown is a simulation of the interaction of two solitons when they are in the same starting position, ie. shows the violation of their linear superposition.

Keywords: solitons, Korteweg-de Vries equation, interaction, linear superposition.

INTRODUCTION

When passing through an optical fiber, the various components of the signal is spread at different rates. This effect (GVD) affect the expansion and distortion of signals. In addition, there are various nonlinear effects that also contribute to the impulses distortion during propagation (damping impulses). This distortion is not a major problem at small distances, but the system for the transmission of signals at large distances, these effects are very detrimental for the correct transmission. In order to avoid, these effects can be to balance with appropriate form of impulses, solitons.

Soliton is the impulse that is characterized by the fact that his form does not change during transmission through a specific environment. Environment in which can survive soliton must be such that the nonlinearity and dispersion of the environment compensate each other [1], [2]. Single-mode fiber with negative group velocity dispersion is one such environment, provided that in the fiber extends momentum sufficient intensity to characteristics nonlinear fibers come to expression. From theory of propagation of optical impulses through the

fiber is well known to affect his form of group velocity dispersion resulting from the material dispersion and waveguide dispersion. Negative group velocity dispersion means that the higher frequency components in the spectrum of impulses transmissions faster than lower frequency components [3], [4], [5]. Nonlinear characteristics of the fibers will appear at high field intensities. We know that the highest density power in the downtown core and to decline to the shell. Where the largest field intensity, the refractive index is greatest. This behavior is known as Kerr effect. The result of this phenomenon is the dependence of the speed of propagation of the signal level. This leads to a current change phase depending on the signal level that is manifested with self-phase modulation impulses. When balanced the effects of negative group velocity dispersion and self-phase modulation due to Kerr effect the impulse given which does not change shape during propagation [6].

Solitons in transmission systems with optical fibers were first predicted theoretically more 1973rd, and the experimentally obtained 1980th years [7]. Since then, many times they are used to explore the basic features of pulse propagation.

Development of computer technology it is possible that the numerical methods to get several different types of equations that give soliton solutions [8]. In this paper we used soliton solutions Korteweg - de Vries nonlinear dispersive equations

SOLITON SOLUTION OF KORTEWEG – DE VRIES EQUATION

Korteweg - de Vries (KdV) equation is one of the most famous nonlinear dispersive equations [9]. KdV equation in standard form as follows:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{\partial^3 u}{\partial x^3} = 0 . \quad (1)$$

KDV equation has a structure in which the dispersive and nonlinear terms can bring into balance, which form a stationary solution.

Consider the individual nature of the nonlinear $\left(u \frac{\partial u}{\partial x}\right)$ and dispersive $\left(\frac{\partial^3 u}{\partial x^3}\right)$ term in equation (1).

Nonlinearity causes a “breaking” waves, which can be seen from the solution of the equation:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = 0 . \quad (2)$$

Peak of the wave narrows and front part becomes steeper, until it comes to breaking.

Dispersion in equation (1) characterized term of the third spatial derivation, and its effect is to expand the wave packets, so that they lose their shape and localization.

That which makes the KDV equation and its solution is important that these two effects, nonlinearity and dispersion exactly compensate, so in the solution KDV equation no breaking waves, while wave packets indefinitely retain their form (no dispersion wave). Spread of the wave preventing to develop peak that would break, while, on the other hand, the nonlinearity, which leads to the development of peak waves, preventing the spread of the waves - the two effects are in very fine balance.

Find the solution KDV equation (1) in the form of traveling waves $u(x, t) = u(x - ct)$, where c is the velocity of propagation of waves along the x -axis. In the derivative we choose integration

constants so that eventually we get soliton solution, based on the conditions for wave function and all its derivatives tend to zero at infinity. Marking phase of the wave functions as $\xi = x - ct$, and how

$$\frac{\partial u}{\partial x} = \frac{d\omega}{d\xi} , \quad \frac{\partial u}{\partial t} = -c \frac{d\omega}{d\xi} , \quad (3)$$

KdV equation partial write a simple differential equation of the variable ξ :

$$(\omega - c) \frac{d\omega}{d\xi} + \frac{d^3 \omega}{d\xi^3} = 0 . \quad (4)$$

Integration and further mathematical calculations we get the basic form of solitons:

$$u(x, t) = 3c \operatorname{sech}^2 \left(\frac{\sqrt{c}}{2} (x - ct) \right) . \quad (5)$$

Solution (5) is shown graphically in Fig. 1, where ξ represents phase $\xi = x - ct$, and ordinate amplitude waves, where the for velocity is taken $c = 1$.

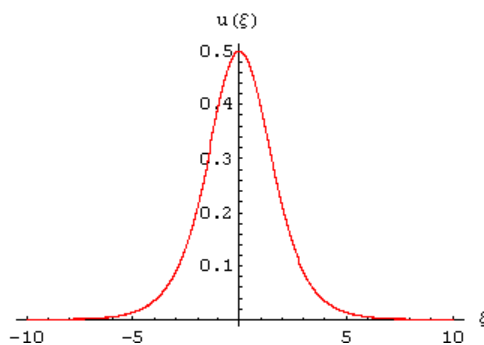


Fig. 1: One-soliton solution of KdV equation for $c = 1$

There are three typical sizes in the solution (5), amplitude ($3c$), width ($2/\sqrt{c}$) and velocity (c) related to each other, and this shows the following properties of soliton:

- amplitude is proportional to the speed
- amplitude is inversely proportional to the square of the width of the wave,
- soliton moving in the positive direction of the x -axis (solution makes sense only for positive velocity).

Faster waves are more localized and have a larger amplitude of the slower.

The last characteristic is formally represented by the root of velocity in the expression (5), and a reflection of the lack of invariance of KdV equation with respect to time inversion $t \rightarrow -t$.

Qualitatively, the form of solitons (5) resembles the Gaussian: a smooth, symmetrical, with a pronounced maximum. However, the similarity stops, because it does not show Gaussian solitons properties. Gaussian does not meet the KdV equation, and if you take the solution KdV equation with a Gaussian as initial condition, then, Gaussian break on the sequence of the form like (5) different amplitude, where the components with the highest amplitude away fastest in accordance with the first above-mentioned property.

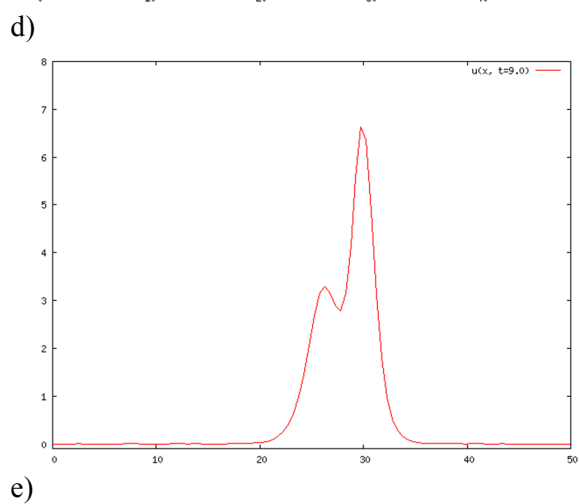
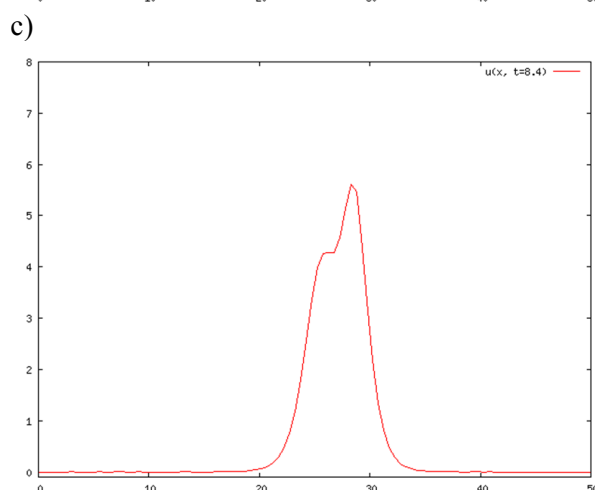
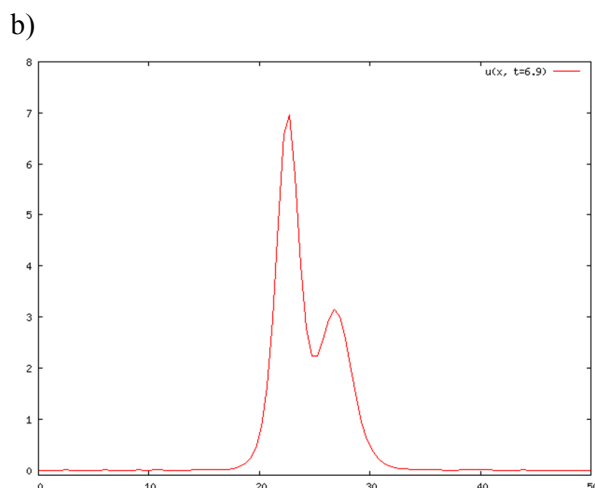
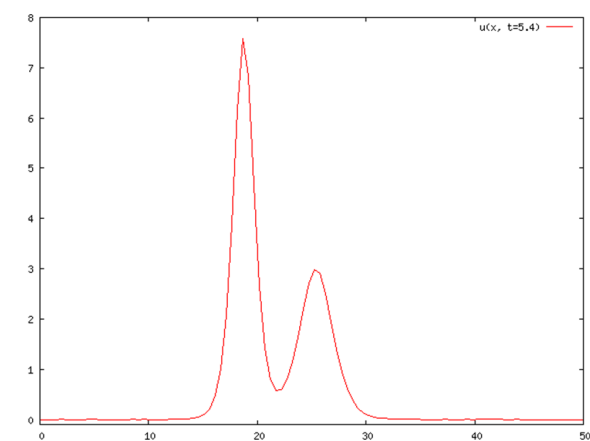
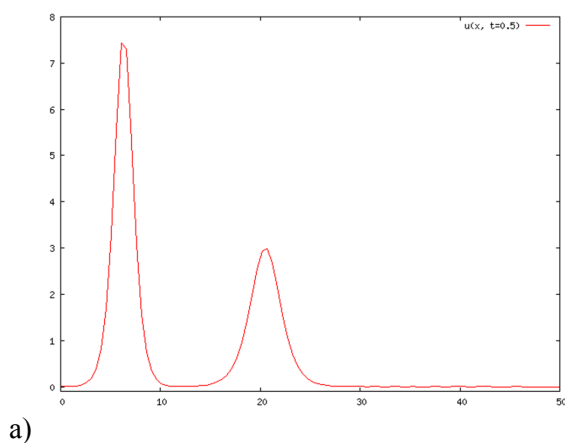
If want to show the movement of waves in space and time, it is necessary to note that the spatial point where the phase is zero ($x - ct = 0$) uniformly moving at a velocity positive x -axis direction.

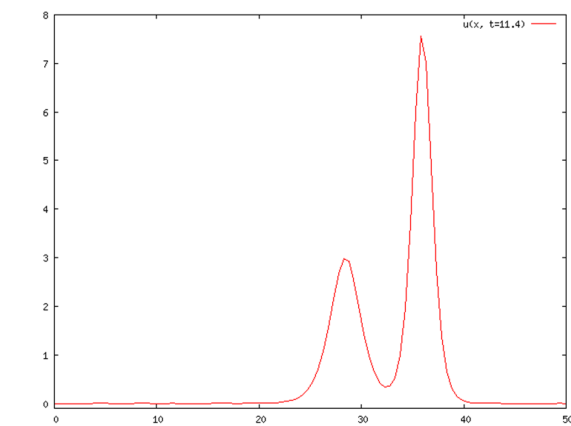
INTERACTION OF TWO SOLITONS

Applying solitons solutions (5) and with mathematical calculation we get an expression for the interaction of two solitons:

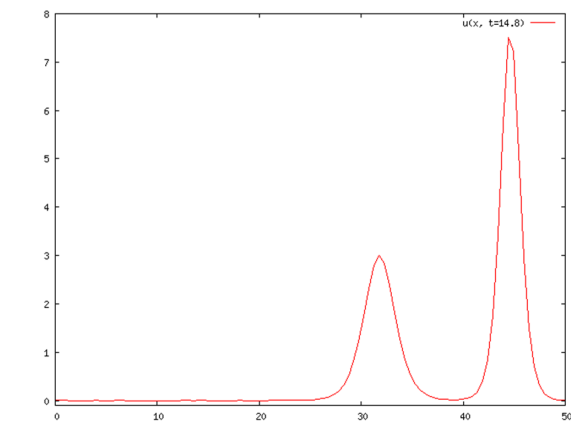
$$u(x, t = 0) = 3c_1 \operatorname{sech}^2 \left(\frac{\sqrt{c_1}}{2} (x - x_1) \right) + 3c_2 \operatorname{sech}^2 \left(\frac{\sqrt{c_2}}{2} (x - x_2) \right) \quad (6)$$

Fig. 2 shows the interaction of two solitons which moving velocity $c_1 = 0,64$ and $c_2 = 0,25$ whose centers is located at the initial time in $x_1 = 5$ and $x_2 = 15$.





f)



g)

Fig 2: Simulation of the interaction of two solitons which moving velocity $c_1 = 0,64$ and $c_2 = 0,25$ whose centers is located at the initial time in $x_1 = 5$ and $x_2 = 15$.

Here is interesting the moment of collision, when nonlinearity is very expressed, and there is a shift in the phase, which is reflected in the "saddle" in Fig. 2.

Detailed consideration in the finer time resolution, observed that this "saddle" at no one moment does not disappear, which is a confirmation of the phase shift.

In this example, in the starting point, there are two solitons package, although relatively close to each other, clearly separated.

At arbitrary initial conditions, next to solitons traveling to the right, and there are traveling waves that spread in the opposite direction.

Previously described properties of solitons occurs only in movement higher-order solitons and is called "breathing" solitons.

This is a negative phenomenon in the movement of solitons.

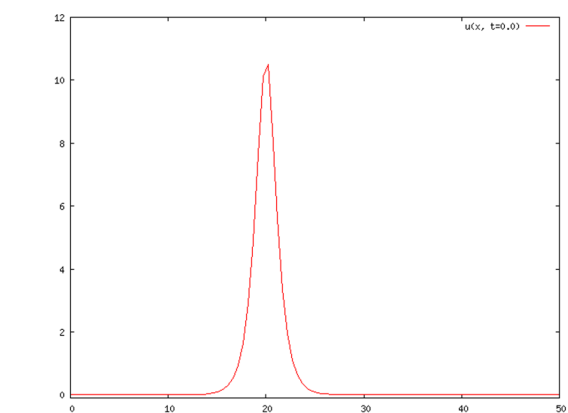
The higher level of solitons it is this phenomenon more pronounced, and easily get to "suffocation" solitons, a phenomenon largely

restricted movement of these waves and therefore the transfer of information.

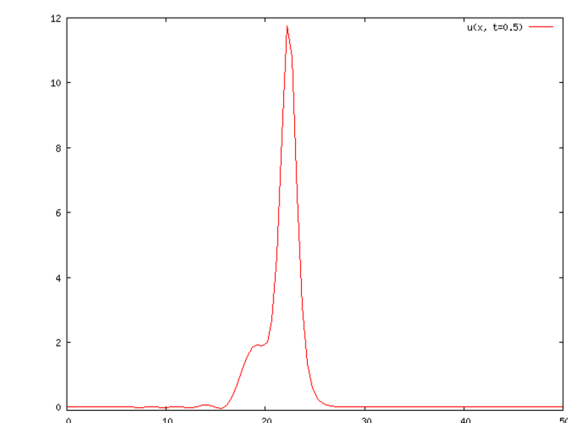
VIOLETION LINEAR SUPERPOSITION OF TWO SOLITONS

Based on the superposition of two solitons (6) was carried out simulations of their collisions in the case when they initial time overlap., $x_1 = x_2$.

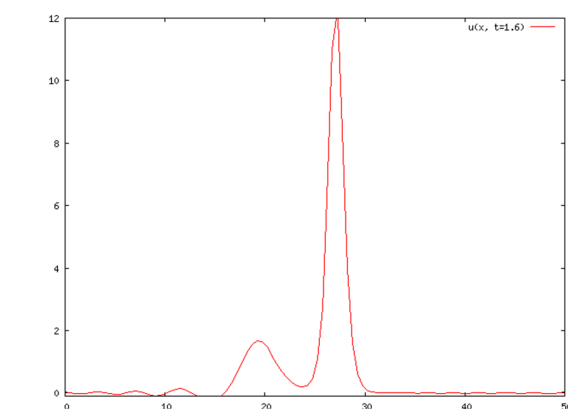
While in the previous case two solitons was at the initial time apart so that one could speak of the approximate linear superposition, we will now consider the violation of linear superposition, where it should be the most visible - at the time of interaction of two solitons, when the nonlinearity reaches its peak (Fig. 3).



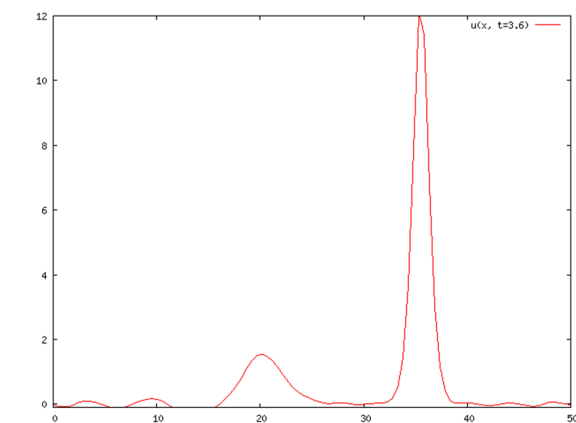
a)



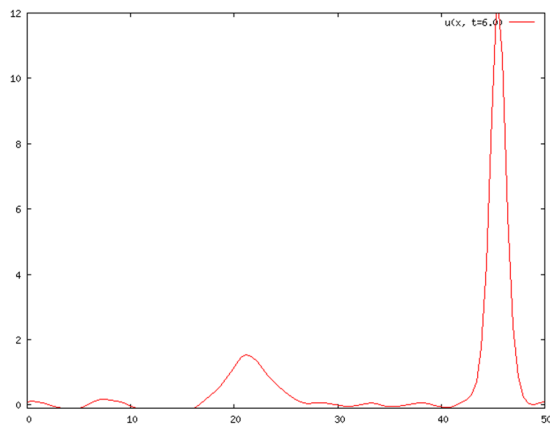
b)



c)



d)



e)

Fig. 3: Simulation of a collision of two solitons in the case when the initial time they overlap, $x_1 = x_2$.

As can be seen from Fig. 3, the expectations are indeed fulfilled, and the nonlinearity is manifested in a number of disorders in addition to the two initial solitons.

There is a change of amplitude and velocity of movement of the two initial solitons, which impairs the quality of transmission and transmission properties of solitons lost that their shape does not change during the transfer.

In addition to the two initial solitons occur and the waves which move opposite to motion the initial solitons, which further undermines the quality of transmission.

CONCLUSION

Presented the analysis and simulation of interaction of two solitons in the transmission path when they are at a certain distance and when they are on the same initial range.

In the first case came to their mutual overlap, but their amplitudes do not change. In the second case leads to changes in velocity and amplitude as well as to the occurrence of other impulse.

One way to avoid the mutual action between two adjacent solitons is to increase the mutual distance between the initial impulses so that $\Delta x = x_1 - x_2 \geq 10$ (for overseas transmission distance of more than 15,000 km).

However, it can be reduced to about $\Delta x \geq 4$ for case where overlapping two solutions no the same amplitude and phase (as is the case in most real systems) due to, for example, random initial chirp from the transmitter, GVD second-order, and random noise amplifiers.

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