

METHOD OF DEFINITION OF THE BASIC PARAMETERS OF THE SAMPLED AND RECONSTRUCTED SIGNALS

Petre Tzv. Petrov

MicroEngineering, Sofia, Bulgaria

Email: ptzvp@yahoo.fr

Abstract

The paper is dealing with the basics of the sampling and reconstruction theory and practice and especially with the evaluation of the sampled and directly reconstructed signal. The approach is based on the principles of the geometry because the simplest electrical test signals could be considered as existing into single plane coordinate system with two perpendicular axes X and Y or axes Time and Amplitude (TA). The method is making possible the evaluation of the differences called "errors" between the corresponding parameters of the original analog test signal and its approximation with rectangular and trapezoidal like staircase function. Examples with sinusoidal signals based on the model "take and memorize until next sample" are given.

Keywords: geometry, simplest test signals, coordinate systems, signal sampling theorems, parameters definitions, errors definition

INTRODUCTION

Evaluation of the parameters of the sampled and reconstructed signals is important for the testing, evaluation, validation and comparison of signal sampling, conversion and reconstruction systems (SSCRS). Frequently, the evaluation is made with sinusoidal and co-sinusoidal signals (SS and CS) produced in XY coordinate system (Fig. 1). The block-diagram of a simplified SSCRS is shown on Fig. 2. The SSCRS has two main paths:

1/ the signal path (SGPT) where the signal is sampled, converted, memorized and reconstructed and

2/ the control path (CNPT) which is controlling all processes in the SSCRS and especially the processes into "signal path".

The signals from both paths should not be mixed. The test equipment is not an obligatory part of the SSCRS. The main goal of the paper is defining and comparison of the basic parameters of the signals in the SGPT under the following principal conditions:

- * The method of direct signal reconstruction is applied (MDSR);

- * The classical geometry in the plane with XY or Time – Amplitude (TA) coordinates system is used. In that coordinate system the signal is sampled, converted, memorized and

reconstructed. This is the simplest case of signal processing.

The definitions of the parameters of the controlled and reproducible analog test signal (ATS) are given and compared with the corresponding parameter of their approximation with rectangular staircase function (RCSF). The parameters of the RCSF (Fig. 3) depend on three basis group of parameters of the SSCRS:

- * Signal sampling factors (SSF) N . Some of them are explained below.

- * Number on the bits of the converters ADC and DAC (n_{adc} and n_{dac} or simply n),

- * The angle of the first sample (φ_0).

The definition of the bandwidth (BW) of the SSCRS is given. The method is intended to evaluate the differences between the sampled original and the reconstructed copy which is related to the quality and safety of the digital samples. It should be noted that the idealized RCSF is a simplified model of the real trapezoidal like function (TRLF) with rounded angles observed after the reconstruction with Digital to Analog Converter (DAC).

THE METHOD AND THE COORDINATE SYSTEM

The proposed method for definitions of the parameters consists of:

* Selecting the appropriate coordinate system where the signal which will be converted and reconstructed.

* Defining the parameters of the ATS and comparing them with the parameters of the corresponding staircase function (SF).

The coordinate system is with two axes in the simplest case (XY coordinate system). Systems with three (XYZ coordinate system) or more axes could be evaluated in similar manner if it is necessary. This is important because the chosen coordinates system determines the number of the independent parameters to reconstruct and the number of the samples to be taken. For example in the XY system the simplest band limited signal (SBLs) with two lines into its spectrum is given with one of the equations below:

$$A = A_m \sin(2\pi ft + \theta) + B \quad (1)$$

$$A = A_m \cos(2\pi ft + \theta) + B \quad (2)$$

The two spectral lines have amplitudes A_m and B respectively. In the XY system SS could have one of the equations below.

$$A = A_m \sin(2\pi ft + \theta) + 0 \quad (3)$$

$$A = A_m \sin(2\pi ft + \theta) + 0 \quad (4)$$

In XYZ coordinate system the SBLs with two frequencies is given with the equation below:

$$A = A_m \sin(2\pi ft + \theta) + B + 0 \quad (5)$$

There are more possibilities for SBLs in XYZ system which will not be exploited here. In XYZ system the SS have one of the equations:

$$A = A_m \sin(2\pi ft + \theta) + 0 + 0 \quad (6)$$

$$A = A_m \sin(2\pi ft + \theta) + 0 + 0 \quad (7)$$

In the cases above the “zeros” are important because they are values of the parameters which must be evaluated and reproduced. In the paper only the signals within XY coordinate system will be examined. But the method is expandable to more than two dimensions.

The parameters of the ATS and the SF to be taken into consideration are extracted from: the shape of the signal, the area covered from the function representing the signal, derivative parameters of the signal (e.g. first derivative, second derivative, etc.) and the errors of the reconstruction associated with each predefined parameter e.g. amplitude errors, phase errors, spectral errors, area errors, time errors, frequency errors etc.

HISTORICAL NOTE

The “sampling theorem of C. E. Shannon” and similar “sampling” theorems are given in [1, 2, 3] are called here “classical sampling theorem” (CST). It is shown that the application of CST to electrical signals is inconsistent because: 1/ It does not give the possibility to evaluate the differences between the sampled and reconstructed signal. 2/ It are demanding infinite number of samples and the application of the function $\sin(x)/x$ which is useless when ATS with known equations are sampled. For evaluation purposes the testing is always made with known signals. 3/ If CST is applied to the analog to digital conversion (ADC) there is no evaluation for any parameter of the digital samples representing the ATS.

It should be noted that in the technical field there are a “**useful conception of infinity**” and “**useless conception on infinity**”. For example using a simplification as “infinity number of bits” is much different that using the concept of “infinity number of samples” or “infinite SSF”. E.g. in the field where $n=10$ bits ($\pm 0.5\text{LSB} = \pm 0.0488\%$) are enough a calculation with $n=20$ bits accuracy could be considered as calculation with infinity number of bits because the error is 1024 times smaller! But if a SS is sampled with SSF $N = 10$ and with $N = 20$ the maximal amplitude errors is decreasing from 4.89% to 1.23% or only 4 times! Consequently asking for infinite number of samples or infinite SSF is too strong to have practical meaning. Please note that as general case increasing SSF N p times is not decreasing the maximal amplitude error $p \cdot p$ times [4].

THE CIRCLE IN THE XY SYSTEM IN PLANE

In Fig. 1 there is a circle in the second quadrant of the XY coordinate system. The coordinate system could be called “Time – Amplitude”. The amplitude could be voltage, current, power or any other appropriate parameter. The vector R (equal to the amplitude, A_m) is making rotating (circular) motion and is drawing a “sine curve” or “sine wave” or more exactly a “sinusoidal signal”. Sometimes term “wave” is used but it is more suitable with three dimensional objects and the term “(electrical) signal” is more useful for two dimensional objects. The parameters and particularities of the circular movement of the radius R are: 1/

$X_0=Y_0=0$ are coordinated of the beginning of the XY coordinates system; 2/ X_1, Y_1 are the coordinates of the center of the circle; 3/ R is equal to the amplitude value A_m of the produced SS; 4/ $\omega = 2\pi f$ is the angular speed of the movement; 5/ The direction of the rotation must be specified (clockwise or anti clockwise). In case of electrical signals it is accepted that the radius is rotating in anticlockwise direction. 6/ X_2, Y_2 are the coordinated of the initial position of the moving radius R . In that case R is starting the movement from horizontal position. 7/ Constant voltage signal (CVSG) is the offset or added constant signal when the electrical analogy is applied. 8/ $2A_m$ is the peak to peak amplitude of the SS and the diameter d of the circle ($d = 2 * A_m$).

Additional parameters are the areas (surfaces) covered by the SS are useful and could be introduced but will not be considered here. After ADC the SS is converted into rectangular or digital staircase function (SF) with parameters different from the parameters of the original SS e.g. amplitudes, area covered by the idealized SF etc.

In fact the circular movement of the radius R is producing the SS component of simples band limited signal (SBLs) with two frequencies into its spectrum and given with one of the equations (1), (2), (3) or (4) depending on the position of the circle and initial position of the radius. If the center of the circle is moved in different point, e.g. on the axis X ($X=0$) or in the beginning of the coordinate system ($X=Y=0$) the number of the parameters to reconstruct are not reduced. From electrical point of view the produced by the vector R analog signals is a SS or a co-sinusoidal signal (CS) with or without offset and with or without initial phase. The equations (1) to (4) have four parameters to reconstruct. The zeros are values of the parameters (not an elimination of the parameters). In order to find all the parameters of the signals above there is a need of at least four samples per period of the SS, not necessarily at equal intervals but at known moments of time. If more that one period is available less than four samples per period are needed.

THE QUALITY AND RELIABILITY OF THE SAMPLES

The questions of quality, safety, and reliability of the samples are important. The most important parameters of the samples are the amplitude and the exact moment of sampling. The

bigger amplitude determines higher accuracy of the signal processing (if any) and higher accuracy of the reconstruction process. The samples are reliable source of information when they have sufficient amplitude. In the ideal case the enough samples must be: 1/ Equal to the amplitudes of the SS (at the points of the maximums and minimums or $(+ A_m)$ and $(- A_m)$; 2/ Much bigger than all errors into the SSCRS; 3/ Much bigger from the least significant bit (LSB) of the ADC; 4/ Samples with stable sampling period and in known moments of time; 5/ Enough samples for recalculations and reconstruction of all parameters of the signal.

The samples with values around zero and comparable to the LSB of the ADC are unreliable and with big relative and absolute errors.

THE BANDWIDTH OF THE SAMPLER

There are three main cases for the “sampler” as the set of:

- * Sample and Hold (SH) and Analog to Digital Converter (ADC),
- * ADC and digital memory for the samples,
- * SH, ADC and digital memory for the samples.

The BW of the sampler could be defined at any appropriate level with the method and formula in [4, 5]. Although the method is illustrated with a SS it is valid for any signal with known equations.

It should be noted that **the lower frequencies in the pass band of the sampler** are sampled with higher sampling factor and have lower amplitude errors. But for these frequencies more undesirable and inevitable products of the sampling, conversion and reconstruction process are entering in the pass band of the reconstruction filter. That is the reason why the quality of the reproduction of the lower frequencies could be not significantly higher that the quality of the reproduction of the higher frequencies. This is especially important when the evaluation is made by humans not by machines.

METHOD OF SAMPLING, CONVERSION AND DIRECT RECONSTRUCTION

Definition: The sampling, conversion and direct reconstruction of the analog signal (SCDRAS) is process of converting of analog signal into digital codes through analog to digital converter (ADC), memorizing the codes and converting the codes into analog forms (in general into

analog trapezoidal staircase function with rounded angles) though digital to analog converter (DAC) practically immediately and without any treatment of the samples.

The paper is applicable to any type of ADC and DAC system, under the condition that the necessary test and control points are available. Fig. 2 is showing block diagram of one implementation of the method SCDRAS. The equipment is including the following principal blocks: 1/ Signal source; 2/ Sampling system (ADC or Sample and hold (SH) plus ADC); 3/ Reconstruction system (DAC), 4/ Digital memory (M) between ADC and DAC; 5/ Digital controller with stable clock source; 6/ The test equipment to compare the ATS and reconstructed signal.

In the simplest case the three definitions below are applicable.

The simplest “sampling subsystem” is given with the “formula” below:

$$\text{SMSS} = \text{ADC} + \text{M} \quad (8)$$

The simplest “reconstructing subsystem” is given with the “formula” below:

$$\text{RCNSS} = \text{M} + \text{DAC} \quad (9)$$

The simplest “sampling and reconstructing system” is given with the “formula” below:

$$\text{SSRNS} = \text{ADC} + \text{M} + \text{DAC} \quad (10)$$

Adding more analog or digital memories for the samples does not change the principle.

SIGNAL SAMPLING FACTOR N

Fig. 3 is showing a SS sampled and converted into rectangular SF (RCSF). The parameters of the SS and RCSF will be defined and compared in order to evaluate the effect of approximation of SS with RCSF.

The most important parameter of the sampling is called signal sampling factors (SSF) N. In fact there are several definitions of SSF depending on the sampled signals. Here it is given definitions of only two of them:

* The SSF for the DC signal (N_{dc}).

* The SSF for the SS (N_{ss}).

Definition: The SSF for the DC signal is equal to one when only that signal is sampled.

One sample is enough to reconstruct a DC according to the model “take and memorize the sample until next sample comes”, or

$$N_{dc} = 1 \quad (11)$$

The error of sampling is determined by the size of $\pm 0.5\text{LSB}$ and the other errors of the

sampling and conversion system. When the DC is part of more complex signal different definition and rule are applied.

Definition: The SSF for a SS signal with zero DC components is given with the equation below

$$N_{ss} = F_d / F_{ss} \quad (12)$$

Where:

F_d is the digitizing (sampling) frequency,

F_{ss} is the frequency of the SS,

By definition $N_{ss} \geq 0$, but in most of the cases $N_{ss} \geq 4$ is selected and in order to reduce the errors much higher SSF is maintained, e.g. $N_{ss} > 24$ is preferred.

Amplitude errors of the sampling are defined in [4, 5].

Example of application of SSF N: The SS is sampled and converted with ADC with infinite number of bits. The maximal deviation from the maximum ($+A_m$) of the SS is:

$$\varphi_{devmax} = 90\text{deg.} - (\varphi_{\text{sampling}})/2 = 90 - (360/N)/2 = 90 - 180/N, \text{ deg} \quad (13)$$

The amplitude of the maximal guaranteed sample is:

$$A_{smax} = A_m \sin(90 - 180/N) \quad (14)$$

The maximal deviation from $+A_m$ (as amplitude) is:

$$E_{max} = 1 - \sin(90 - 180/N) = 1 - \cos(180/N) \quad (15)$$

The calculations above are giving guarantee for the amplitude and quality of the samples obtained during the test of the conversion system with SS.

THE METHOD OF SAMPLES EVALUATION FOR ANY KNOWN SIGNAL

The approach from above is applicable to any signal with known equation in the following steps:

* Infinite (enough) number of coding bits;

* The point of the maximum of the signal is A_{smax} in moment T_{smax} ;

* The two deviations from that points, called left and right deviation are calculated A_{slmax} and A_{srmax} . Each deviation is calculated at moment $+Ts/2$ (A_{sr}) and $-Ts/2$ (A_{sl}), where Ts is the sampling period,

$$A_{slmax} = A_{smax} - A_{sl} \quad (16)$$

$$A_{srmax} = A_{smax} - A_{sr} \quad (17)$$

* The intervals from the left and right side of the maximum T_{max} or $[T_{lmax}, T_{max}]$ and $[T_{max}, T_{rmax}]$ are analyzed in order to be sure that no greater deviation from the maximum of

the signal is possible due to the movement of the sampling points.

* The steps above are applied to the minimums of the signal.

* In order to find the maximum deviation from peak to peak amplitude of the signal the maximal difference from the peak to peak amplitude of the signal A_{sspp} and the peak to peak amplitude of the SF A_{sspp} is calculated:

$$A_{ssfpp} = A_{sspp} - A_{sfpp} \quad (18)$$

Fig. 3 is showing a particular case of sampling and direct reconstruction of SS. The main parameters of the figure are: 1/ SSF $N = 3$ or there are three samples per period of the SS; 2/ Angle of the first sample $\varphi_0 = 60$ degrees; 3/ Sampling angle $\varphi_d = 120$ degrees (regular sampling, $\varphi_d = 360/N$); 4/ Sufficient coding bits n .

In order to simplify the diagram a DC component and the initial phase of the SS are shown as zeros in the Fig. 3.

BASIC PARAMETERS OF THE TEST SIGNALS

The parameters of the ATS are known before each test. This is important because these parameters are compared with the parameters of the reconstructed signal. For example the “root-mean-square” (rms) value of the SS is given with the formula below:

$$A_{rms} = A_m * \sqrt{2} = 0.707 * A_m \quad (19)$$

Also the average value (A_{avg}) of SS is calculated by taking the corresponding parameter in a suitable interval. If the full cycle of the SS is taken then

$$A_{avg} = 0 \quad (20)$$

But if only the first 90 degrees are taken into consideration then:

$$A_{avg90} = 2 * A_m / \pi = 0.636 * A_m \quad (21)$$

For the needs of the signal sampling and reconstruction more detailed definitions are needed and parameters should be defined for a part of the cycle of the sampled signal. The basic parameters defined for that purpose are listed below and some of them are shown on the Fig. 1 and Fig. 3: A_{ssmp} is the positive peak value of the SS; A_{ssmn} is the negative peak value of the SS; A_{sspp} is the peak to peak value of the SS.

Since in the general case the ATS is not symmetrical the “absolute signal amplitude asymmetry” (ASAA) should be defined:

$$ASAA = A_{ssmp} - A_{ssmn} \quad (22)$$

The asymmetry is important parameter of the signal and should be reconstructed. The “signal amplitude asymmetry factor” (SAAF) is calculated:

$$SAAF = A_{ssmp} / A_{ssmn} \quad (23)$$

The MSRSS - maximal slew rate of the sampled signal is also calculated according the signal equation.

Some more basic parameters to define and control are:

T_{ss} - period of the sampled SS

F_{ss} - frequency of the sampled SS

$$F_{ss} = 1 / T_{ss} \quad (24)$$

The parameters of area covered by the function describing the signal should be defined. Examples are given below, e.g.:

S_{ssp} - area of the positive part of the SS;

S_{ssn} - area of the negative part of the SS;

S_{sst} - total area in one period of the SS;

Detailed definitions of all relevant parameters of the sampled signal and reconstructed signal are important in order to evaluate the effects of the signal sampling and reconstruction. All parameters of the original signal are influenced by the process in SSCRS.

BASIC PARAMETERS OF THE STAIRCASE FUNCTIONS APPROXIMATING THE SIMPLEST TEST SIGNALS

The parameters of the RCSF are used as approximation of the sampled signal. The letters “sf” from “staircase function” is part from any abbreviation concerning the SF. List of some of the parameters is given below:

A_{sfmp} = positive peak value of the sample (zero to positive peak),

A_{sfmn} = negative peak value of the sample (zero to negative peak),

A_{sfpp} = peak to peak value of the staircase function (peak to peak value of the samples),

S_{sfp} - area of the positive part of the SF;

S_{sfn} - area of the negative part of the SF;

S_{sft} - total area in one period of the SF;

S_{sfas} - area asymmetry of the SF

$$S_{sfas} = S_{sfp} - S_{sfn} \quad (25)$$

S_{sfas} - factor of the area asymmetry of the SF:

$$S_{sfas} = S_{sfp} / S_{sfn} \quad (26)$$

Factor of the signal amplitude asymmetry of the SF:

$$FSAA_{sf} = A_{sfmp} / A_{sfmn} \quad (27)$$

$SR_{\max sf}$ - maximal slew rate (rate of change, first derivative) of the SF,

T_{sf} - period of the SF,

F_{sf} - frequency of the SF,

$RT_{\max sf}$ - maximal rise time of the SF,

$FT_{\max sf}$ - maximal fall time of the SF,

$RTTA_{sf}$ - rise and fall time asymetry of the SF

$RTTA = Tr/Tf$, [-] (28)

A_{ovsf} - amplitude of the overshoots (is any) of the SF,

A_{unsf} - amplitude of the undershoots (is any) of the SF,

ϕ_{dsf} - phase angle of the delay of the SF,

A_{slsf} - change of the level of the steps of the SF or slope of the level (ideally no change of the level of the steps),

A_{sfp} - area of the positive part of the SF,

A_{sfm} - area of the negative part of the SF,

A_{sft} - total area in one period of for the time of its existence of the SF,

R_{asf} - radius of the arc of the “rounded angles” of the SF, in case the angles are not ideal,

TLH_{sf} - total level of the harmonics in the SF,

$TALF_{sf}$ - total level of the added frequencies in the SF compared to the ATS

$AOSS_{sf}$ - amplitude of the original SS in the spectrum of the SF.

More parameters could be defined and calculated. E.g. the list could be continued with calculating DC offsets, root-mean-square value, mean values and other moments or for some period values of the signals.

DEFINING DIFFERENCES BETWEEN THE BASIC PARAMETERS OF THE TEST SIGNALS AND THEIR STAIRCASE FUNCTIONS

Differences between the parameters of the ATS and the ST are defined as absolute and relative errors for the corresponding parameter. For example several definitions are given below:

E_p = error in the positive peak values of the samples. This is the difference between the positive amplitude of the ATS and the amplitude of the maximal positive sample:

$$E_p = +A_{ssmp} - A_{spmp} \quad (29)$$

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Similarly E_n is the error in the negative peak value of the samples:

$$E_n = -A_{ssmn} - (-A_{spmn}) \quad (30)$$

The signs in the formulas should correspond to the real situation under evaluation.

In the same manner the difference between the peak to peak value of the signal and peak to peak value of the samples E_{pp} is:

$$E_p = A_{sspp} - \text{abs}(A_{sfpp}) \quad (31)$$

Defining the differences as absolute and relative value is considered straightforward process and will not be made for all differences.

EXAMPLE

The Table 1 is giving an example with comparison of several basic parameters of SS and SF with SSF $N = 2, 3$ and $N=4$ with infinite number of bits n and with several values of the angle of the first sample ϕ_0 . The T_{tcov} - the total conversion time is the conversion time of the ADC, plus transfer time, plus DAC conversion time plus filter setting time plus output setting time is neglected in that case.

Initial conditions for the Table 1 are:

1/ SSN $N=2, 3$, and 4; 2/ angle of the first sample $\phi_0 = 0, 45$ or 90deg ; 3/ $n > 12$ bits for the converter.

The signal is given with the equation:

$$A = A_m \cos(2\pi ft + \theta) + B \quad (32)$$

The calculations are made with the following assumptions: $A_m = 1, \theta = 0, B = 0, T = 1\text{s}, F = 1\text{Hz}$.

The parameters in Table 1 are calculated for a single period of the SS component. The results from the calculations depend on the initial state of the sampler and could be different from period to period. This is the effect of the “parasitic modulation” in the sampler. In that case the initial value of the sampler is zero (value after “reset” condition in most of the cases).

Calculated parameters: Only a small part of the parameters of the SS and the SF are given due to limited the size of the paper.

The parameters of the SS in Table 1 are:

A_{ssmp} is the positive peak value of the sampled SS,

A_{ssmn} is the negative peak value of the sampled SS,

A_{pp} is the peak to peak value of the sampled SS,

The parameters of the SF in Table 1 are:

A_{sfmp} is the positive peak value of the SF,

A_{sfmn} is the negative peak value of the SF,

A_{ffpp} is the peak to peak value of the SF,

S_{sfmp} is the area of the SF above the axis X (the “positive area”)

S_{sfpp} is the total area covered by the SF:
 $S_{sfpp} = \text{abs}(S_{sfmp}) + \text{abs}(S_{sfnp})$ (33)
 The difference between the areas of the SF above and below the axis X is:

Delay in starting the non-zero value of the SF is $D_{\text{sffmax}} = T_{\text{sas}} - T_{\text{sff}} = \varphi_d = 360/N \text{ deg}$ (35)

Where T_{sas} – is the starting time of the sampled analog signal (the moment of the first non zero value),

A full period is considered between φ_0 of the current period until φ_0 of the next period. If that condition is changed the contents of the Table 1 will change too.

Example with calculated basic parameters of the RCSF

SSF	2	2	2	3	4	4
n	∞	∞	∞	∞	∞	∞
φ_0	0	45	90	0	0	45
A_{ssmp}	1	1	1	1	1	1
A_{ssmn}	1	1	1	1	1	1
A_{sspp}	2	2	2	2	2	2
A_{sfmp}	0	0.707	1	0.866	1	0.707
A_{sfmn}	0	-0.707	1	-0.866	-1	-0.707
A_{sfpp}	0	1.414	2	1.732	2	1.414
S_{sfmp}	0	0.354	.5	0.433	0.25	0.177
S_{sfmn}	0	0.354	.5	0.433	0.25	0.177
S_{sfpp}	0	0.707	1	0.866	0.50	0.353
D_{sfmax}	π	π	π	$2\pi/3$	$\pi/2$	$\pi/2$

The proposed method is applicable for definitions and calculations of the parameters and

any known sampled signal and the approximating SF in the selected coordinate system. The method is making possible the calculation of the main factors of the sampling and conversion process in order to obtain the targeted value of the parameters. The sampling, conversion and reconstruction errors are inevitable and the ATS cannot be reconstructed completely. Consequently, the analog processing should be applied whenever is possible due to the destructive and noisy nature of the SSCRS and the digital processing.

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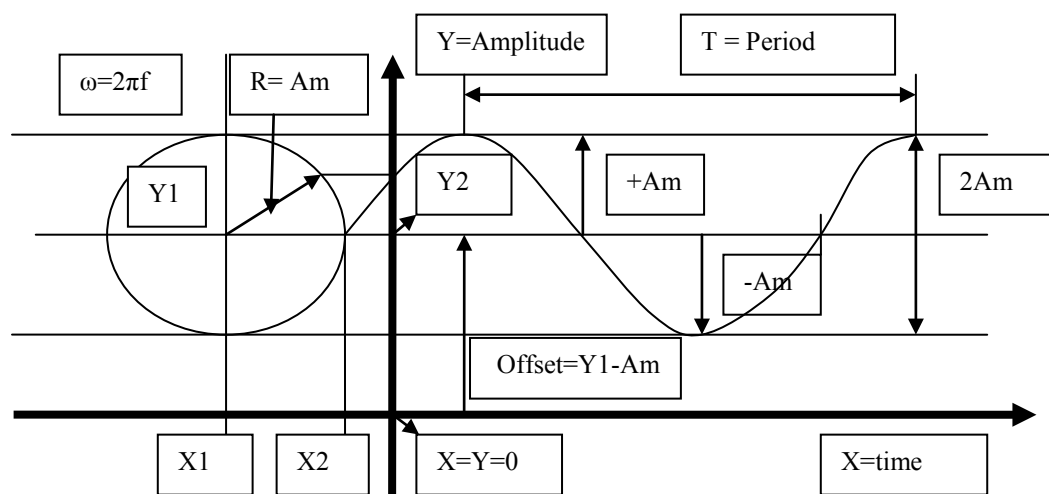


Fig. 1. The rotating vector in second quadrant (and its basic coordinates) is producing a sinusoidal signal

