

РАЗПРЕДЕЛЕНИЕ НА НАПРЕЖЕНИЕТО ОКОЛО ЦЕНТРАЛНО РАЗПОЛОЖЕНИ ОТВОРИ В МЕТАЛНИ ПЛАНКИ, ПОЛУЧЕНО С МКЕ

STRESS DISTRIBUTION AROUND CENTRALLY LOCATED HOLES IN PLATES OBTAINED BY THE FEM

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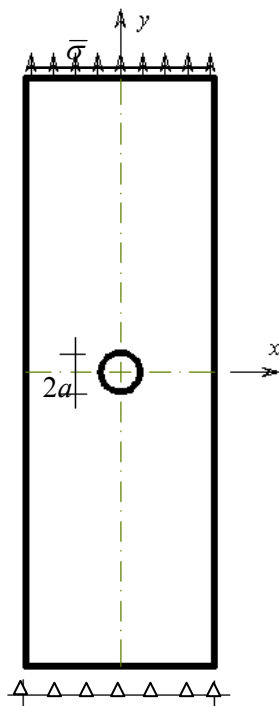
Abstract

This paper presents the results concerning the stress distribution around centrally located holes, calculated by FEM models. The domain of the models is discretized by 2D plane finite elements using ANSYS Structural code. A practically applicable graph of the stress concentration factor (SCF) as a function of the ratio hole diameter to the plate width is finally obtained. The well-known theoretical solution, related to infinite plate is compared with the numerical one.

Keywords: plates with holes, stress concentration factor (SCF), the finite element method (FEM).

INTRI

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A practically applicable graph is the stress concentration factor (SCF) versus the ratio of the hole diameter to the plate width. Such curves can be found in design handbooks [1]. The concentration factors are based on the gross or net sections. Symbolically, the relation between these factors can be given as

$$SCF_g = SCF_t \left(\frac{w}{w - D} \right).$$

The aim of the present study is graph of the SCF to be obtained by FEM models and the numerical results to be compared with the theoretical solutions, related to infinite plates. The results in this work are shown to SCF_g .

THEORETICAL SOLUTION

The case of small circular hole in wide plate under uniform unidirectional tension offers illustration of localized stress redistribution [2]. The theory of elasticity solution is exp-

where
net area
is the total applied load; w - the width of the plate, and t is the thickness, fig. 1. In the same figure the diameter D is denoted as $2a$.

ressed in terms of a radial stress σ_r , a tangential stress σ_θ and a shearing stress $\sigma_{r\theta}$, see *fig. 2*.

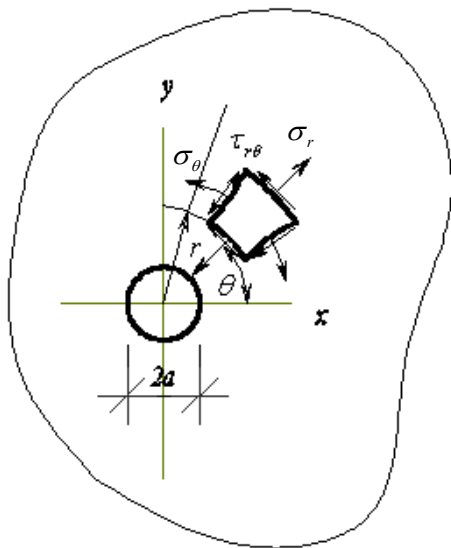


Fig. 2. Illustration of stress distribution in polar coordinate system

The equations, by means of which these stresses can be calculated, are

$$\begin{aligned}\sigma_r &= \frac{\bar{\sigma}}{2} \left(1 - \frac{a^2}{r^2} \right) + \frac{\bar{\sigma}}{2} \left(1 - \frac{4a^2}{r^2} + \frac{3a^4}{r^4} \right) \cos 2\theta \\ \sigma_\theta &= \frac{\bar{\sigma}}{2} \left(1 + \frac{a^2}{r^2} \right) + \frac{\bar{\sigma}}{2} \left(1 + \frac{3a^4}{r^4} \right) \cos 2\theta\end{aligned}\quad (2)$$

$$\sigma_{r\theta} = -\frac{\bar{\sigma}}{2} \left(1 + \frac{2a^2}{r^2} - \frac{3a^4}{r^4} \right) \sin 2\theta$$

$$\sigma_{r\theta} = \sigma_{\theta r}$$

where $\bar{\sigma}$ is the magnitude of the applied uniaxial tension [2]. Obviously, the above relations predetermine an exponential form of attenuation of the stress distribution.

The critical plate section is located on line $y = 0$ in Cartesian coordinates, see *fig. 1*, or line $\theta = 0$ and $\theta = \pi \text{ rad}$ in polar coordinates, see *fig. 2*. Then eqs. (2) can be simplified as

$$\sigma_r = \frac{\bar{\sigma}}{2} \left(1 - \frac{a^2}{r^2} \right) + \frac{\bar{\sigma}}{2} \left(1 - \frac{4a^2}{r^2} + \frac{3a^4}{r^4} \right)$$

$$\sigma_\theta = \frac{\bar{\sigma}}{2} \left(1 + \frac{a^2}{r^2} \right) + \frac{\bar{\sigma}}{2} \left(1 + \frac{3a^4}{r^4} \right) \quad (3)$$

$$\sigma_{r\theta} = -\frac{\bar{\sigma}}{2} \left(1 + \frac{2a^2}{r^2} - \frac{3a^4}{r^4} \right)$$

$$\sigma_{r\theta} = \sigma_{\theta r},$$

and $\sigma_r \equiv \sigma_x$, $\sigma_\theta \equiv \sigma_y$.

When $\theta = 0$ or $\theta = \pi \text{ rad}$ and $r = a$ the maximum tangential stress is $\sigma_\theta^{\max} = 3\bar{\sigma}$. If $r \rightarrow \infty$ then $\sigma_\theta = \bar{\sigma}$. Obviously, when $a = 0$ then $\sigma_\theta^{\max} = \bar{\sigma}$.

On the boundary of the hole ($r = a$) eqs. (2) can be reduced to

$$\sigma_r = 0$$

$$\sigma_\theta = \bar{\sigma} (1 + 2 \cos 2\theta) \quad (4)$$

$$\sigma_{r\theta} = 0$$

$$\sigma_{\theta r} = 0.$$

Analytical solution of the stress distribution of infinite plate with central elliptical hole is given in [3]. Stress $\sigma_\theta^{\max}$ can be calculated from

$$\sigma_\theta^{\max} = \bar{\sigma} \frac{1 - m^2 + 2m \cos 2\alpha - 2 \cos 2(\theta - \alpha)}{1 - 2m \cos 2\theta + m^2} \quad (5)$$

where α is the angle between axis x and the load direction, and m ($0 \leq m < 1$) is the Pascal parameter. In the case of circle hole $m = 0$ and $\sigma_\theta^{\max} = 3\bar{\sigma}$.

FINITE ELEMENT MODEL

The plane finite element *PLANE42* has been used for 2D modelling of the plate. The element is defined by four nodes having two degrees of freedom at each node: translations in the nodal x and y directions. The element input data include thickness (for the plane stress option only) of the nodes. The geometry

of the element in an isoparametric form [4] is illustrated in *fig. 3*.

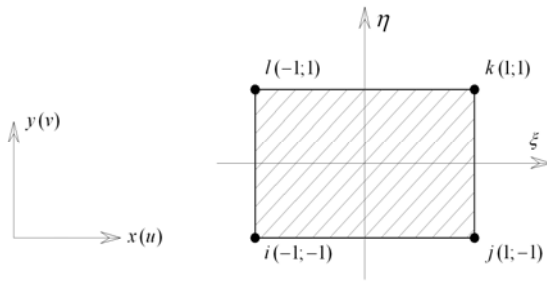


Fig. 3. Geometry of the PLANE42 finite element

The assumed material properties are $E = 20e7 \text{ kN/m}^2$ and $\nu = 0.3$. The imposed boundary conditions are: (i) $u_x = 0$ on all the contour lines; (ii) $u_y = 0$ on the bottom line; The applied loading is $F = 500 \text{ kN}$ acting on the upper line. It is assumed a constant value for the width of the plate $w = 0.21 \text{ m}$. The plate thickness is 0.02 m . Then $\bar{\sigma}$ and $\sigma_{\theta}^{\max}$, using (3), are respectively

$$\bar{\sigma} = \frac{F}{wt} = \frac{500}{0.21 \times 0.02} = 119048 \text{ kN/m}^2$$

and

$$\sigma_y^{\max} \equiv \sigma_{\theta}^{\max} = 3\bar{\sigma} = 357143 \text{ kN/m}^2.$$

The hole diameter range varies from 0.02 m to 0.12 m , see *tabl. 1*.

Figure 4 illustrates the finite element mesh configuration in neighbourhood of the hole.

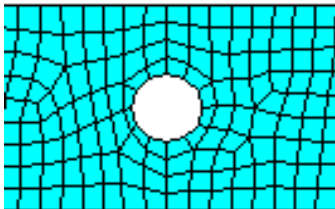


Fig. 4. Finite element mesh around the hole

RESULTS

Figures 5,6,7 show the obtained numerical results.

The values of SCF_g for the different hole diameter can be seen in *Table 1*.

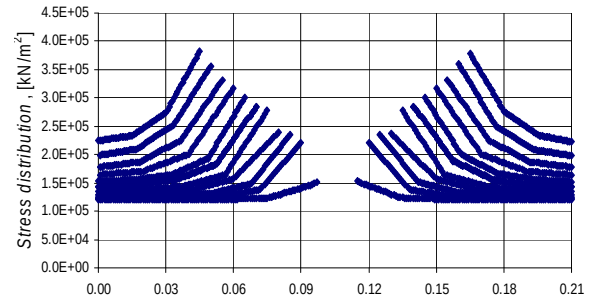


Fig. 5. Stress distribution (σ_y) on line $y = 0$

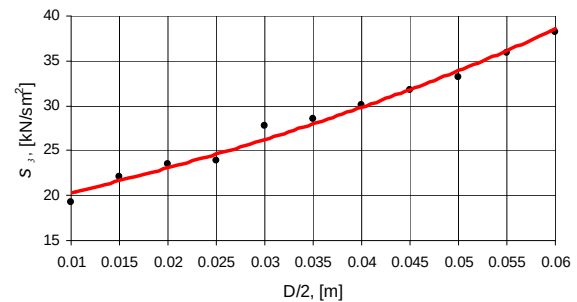


Fig. 6. Maximum stress σ_3 as a function of the hole radius

Table 1. SCF_g results

No	D [m]	D/w	SCF_g
1	0.02	0.095238	1.28
2	0.03	0.142857	1.86
3	0.04	0.190476	1.98
4	0.05	0.238095	2.00
5	0.06	0.285714	2.33
6	0.07	0.333333	2.39
7	0.08	0.380952	2.52
8	0.09	0.428571	2.67
9	0.10	0.476190	2.78
10	0.11	0.523810	3.02
11	0.12	0.571429	3.20

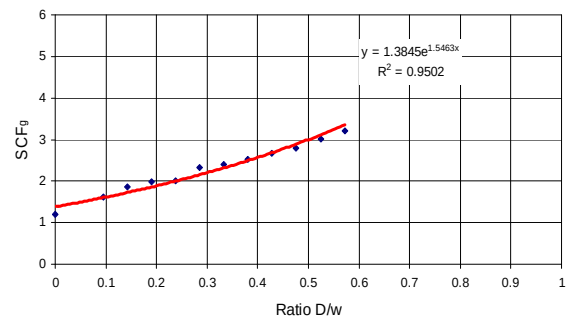


Fig. 7. Stress concentration factor as a function of the ratio hole diameter to plate width

CONCLUSION

The FEM computational technology gives new opportunity the stress distribution around holes in plates to be obtained in more accuracy and the stress concentration factor to be calculated. Such problems often are met in the engineering practice. Obviously, if the hole diameter is very small number, say $\varepsilon \ll 1$, then the maximum stress, calculated by eq. (2) always leads to $\sigma_{\theta}^{\max} = 3\bar{\sigma}$. However, the equilibrium $\sum V = 0$ can be satisfied only if the attenuation of the stress σ_{θ} on line $y = 0$ is given by very high degree. But the attenuation degree in eq. (2) is independent of the hole diameters.

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