

FORCE EXCITATION BY RANDOM NOISE AND IMPACT

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Abstract

Direct application of force excitation for instant frequency response function is discussed. For mobility measurements the structure must be excited by a measurable dynamic force. Here the term random applies to the amplitude of the excitation force which, in statistical terms, has a normal or Gaussian probability distribution. On average however, the spectrum is flat and continuous, containing energy at approximately the same level for all frequencies. Due to the random characteristic of the signal, the structure is excited over a wide force range at each frequency. This randomizes any nonlinear effects, and averaging then gives a best linear approximation. The most popular excitation technique used for modal analysis is impact, or hammer excitation. The waveform produced by an impact is a transient (short duration) energy transfer event. The spectrum is continuous, with a maximum amplitude at 0 Hz and decaying amplitude with increasing frequency.

Advantage and disadvantage of both techniques are discussed. Results from real measurements are presented in graphs.

Keywords: force excitation, input, output, test structure, free-condition, grounded-condition.

INTRODUCTION

It is appropriate to consider vibration methods in general in order to involve the both practical procedures: force excitation by random noise and impact. In the last decade the vibration measurement was preferred both input (force excitation) and output (response). It can be concerned on the type of measurement, where both excitation and response are measured simultaneously so that the basic equation can be used to deduce the system properties directly from the measured data. There are a number of different approaches which can be adopted.

The present study concentrates on one which refers to as the “single-point- excitation” method [1], [2], [7]. When large structures are concerned, the measuring system involves simultaneous excitation at several points. The interest will be focused on the more straightforward approach, where the excitation is applied at a single point. In matter in fact this point may be fixed at measuring points of the structure.

For a truly excitation, a different approach is required although it is possible to undertake such measurement using the same spectrum analyzer. Often, the source of a random command signal is found in an external device such as noise generator and not in the analyzer itself,

although some types do contain independent noise sources for the purpose.

The term random applied to the amplitude of the excitation forces, which in statistical terms has normal or probability distribution of *Gauß* such a type of excitation [1], [3], individual time records in the analyzer contain data with random amplitude and phase at each frequency. On average however, the spectrum is flat and continuous, containing energy at approximately the same level for all frequencies.

Due to the random characteristic of the signal the structure under test is excited over a wide force range at each frequency. This randomizes any nonlinear effects, and averaging then gives a best linear approximation.

EXPOSITION

1. Structure preparation

Regardless the shape of the structure, the type of physical support during measurement is a matter of great concern. The first think to do is whether the structure under test is to be tested in a “free” condition or “grounded” [3], [4], [5]. Free condition means the test object is not attached to the ground at any of its coordinate and is in effect, freely suspended in space, presenting a low frequency system. That must be condition where the structure can

exhibit rigid body modes, which are determined solely by its mass and inertia properties and in which there is not bending or flexing at all. In practice it is not acceptable to provide a “really” free support to provide a perfect suspension system which closely approximates to this condition. This can be achieved by supporting the test piece on very soft springs such as might be provided by light elastic bands so that the rigid body modes, while no longer having zero natural frequencies, have values which are very low in relation to those of the bending modes. That “very low” in this context means that the highest “rigid-body” mode frequency is less than 12-18% of that for the lowest bending mode.

If it can be achieved a suspension system of this type, then it will still derive the rigid body properties from the very low frequency behavior of the structure influence on the flexural modes that are the object of the test. A different precaution which can be taken to ensure minimum inertness by the suspension on the lowest bending mode of the structure, the most vulnerable, is to attach the suspension as close as possible to nodal points of the mode in question. Particular attention should be paid to the possibility of the suspension adding significant damping to otherwise lightly damped test-pieces

It is necessary to note that any rigid body will possess no less than 6 modes and it is also necessary to check that the natural frequencies of all of these are sufficiently low before being satisfied that the suspension system used is sufficiently soft. To this end, suspension wires etc. should generally be normal to the primary direction of vibration.

The other type of support is referred to as “grounded” because it attempts to fix selected points on the structure to ground. While this condition is extremely easy to apply in theoretical analysis, simply by deleting the appropriate coordinates. The reason for that it is very difficult to provide a base or foundation on which to attach the test structure which is sufficiently rigid to provide the necessary grounding. All structures have a finite impedance (non-zero-mobility) and thus cannot be regarded as truly rigid but whereas one is able to approximate the grounded condition without taking

extraordinary precautions when, designing the support structure.

2. Random Vibration

For a real force excitation, a different approach is required although it is possible to undertake, such measurements using the same spectrum analyzer. Often, the source of a random command signal is found in an external device such as noise generator and not in the analyzer itself, although some types do contain independent noise sources for the purpose. In the last years different types of signals have been generated in the LabVIEW 8 *Student Edition* from National Instruments, Inc, using standard power amplifier.

The term random applied to the amplitude of the excitation force, which in statistical terms has a normal or probability distribution of *Gauß*. Such a type of excitation, individual time records in the analyzer contain data with random amplitude and phase, at each frequency. On average however, the spectrum is flat and continuous, containing energy at the same level for all frequencies. Due to the random characteristic of the signal, the structure is excited over a wide force range at each frequency. This randomizes any nonlinear effects, and averaging then gives a best linear approximation. Using the complex type of vibration, both excitation and response are characterized by random processes. As a result, neither excitation nor response signals can be subjected to the known calculation of *Fourier* Transform. Basic parameters of random signal description are presented. As a matter in fact, introducing two sets of parameters, which are used to describe random signals, one based in the time domain, referred to as Correlation Functions and the other the Spectral Density. Consider a typical random vibration parameter $F(t)$, presented in Fig. 1.

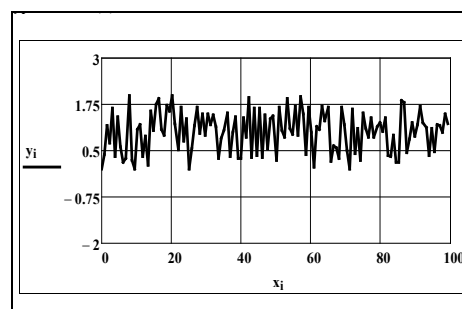


Fig. 1. Random signal, time history

If the Autocorrelation Function $R_{FF}(\tau)$ is the expected and average value of the product $[F(t)F(t+\tau)]$, computed along the time axis leads to:

$$R_{FF}(\tau) = E[F(t)F(t+\tau)] \quad (1)$$

This will always be a real and even function of time and it could be generally take the form (Fig. 2).

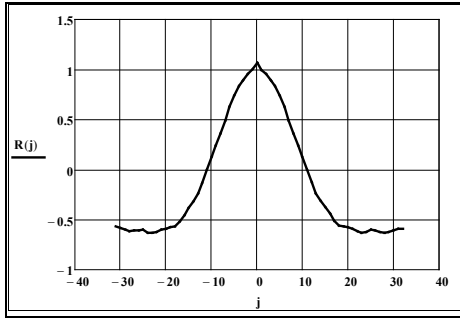


Fig. 2. Autocorrelation Function

The correlation function, unlike the original quantity $F(t)$ does satisfy the requirements for *Fourier Transformation*. Thus one can obtain its *Fourier Transform* by the usual equation.

The resulting parameter is called a Spectral Density. In this case the Auto-Power Spectral Density (PSD) is defined as:

$$S_{FF}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} R_{FF}(\tau) e^{-j\omega\tau} d\tau \quad (2)$$

The Auto-Spectral Density is real and even functions of frequency and does in fact provide a frequency description of the original function $F(t)$. It has units of F^2/ω and would generally appear as in the plot of Fig. 3.

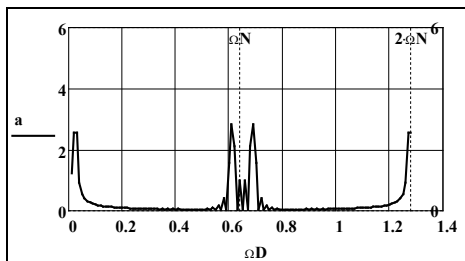


Fig. 3. Power Spectral Density

A similar concept can be applied to a pair of functions as $F(t)$ and $X(t)$. Both functions are usually used to produce cross-correlation and cross-spectral density functions. The produced cross-correlation function $R_{XF}(\tau)$ is defined as:

$$R_{XF}(\tau) = E[X(t)F(t+\tau)] \quad (3)$$

and the cross-spectral density (CSD) is presented as its *Fourier Transform*:

$$S_{XF} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} R_{XF}(\tau) e^{-j\omega\tau} d\tau \quad (4)$$

Cross –correlation functions are real, but not always even, functions of time and cross spectral densities, unlike auto-spectral densities, are generally complex functions of frequency with the particular conjugate property that:

$$S_{XF}(\omega) = S_{FX}^*(\omega) \quad (5)$$

Already established the necessary parameters to describe random processes, one is in position to define the input/output relationships for systems undergoing random vibrations. After derivation the final equation, which allows the calculation of response from known excitations, one should not present a full analysis and can indicate the main way followed without detailing all the mathematics. In this way, it can demonstrate the basis of the analysis and the origin of the final expressions, which are the only ones required in normal modal testing practice.

Consider the time domain, the analysis of total response of the system will be given by adding and superimposing all the incremental responses are:

$$X(t) = \int_{-\infty}^{+\infty} H(t-t') F(t') dt' \quad (6)$$

It is possible to derive an expression for $X(t)$, respectively $X(t+\tau)$. Finally, calculating the response auto-correlation $R_{XX}(\tau)$ can be written as:

$$R_{XX}(\tau) = E[X(t)X(t+\tau)] \quad (7)$$

The equation (7) can be transformed to describe the response auto-correlation in terms of the excitation $R_{FF}(\tau)$. The Eq. (7) can also be transformed in the form:

$$S_{XX}(\omega) = |H(\omega)|^2 S_{FF}(\omega), \quad (8)$$

where $H(\omega)$ is so kaled estimator.

The Eq. (8) is relatively simple, but it does not cover a complete description of the random vibration condition. Also, it could not be used to satisfy the condition to determine the Frequency Response Function (FRF) from

measurements of excitation and response, containing only the modulus of $H(\omega)$, the phase information is lost in this formula. By introducing a second equation may be generated by similar analysis, based on the cross-correlation between the excitation and the response, the frequency domain of which is:

The estimator H_1 can be seen that is equal to the Cross Spectrum, between the response and the excitation and the frequency domain form of which is:

$$S_{FX}(\omega) = H_1(\omega) \cdot S_{FF}(\omega) \quad (9)$$

For the measurement, the test structure is suspended by some means. The force signal is measured by a force transducer, directly connected at the point where the force is applied. It can therefore characterize this typical measurement as having *noise in the measured output signal*.

The terms Auto-spectrum and Cross Spectrum are describe in the case of dual-channel analyzer.

For the *noise in the input* measurement at its natural frequencies the structure becomes very compliant, which results in high vibration amplitudes.

The situation can be characterized as having noise in the input. The estimator H_2 , which minimizes this noise effect, is:

$$S_{XX}(\omega) = H_2(\omega) \cdot S_{XF}(\omega) \quad (10)$$

Using the final results, Eq. (9) and Eq. (10) an experimental procedure has been adopted to obtain solution of rubber samples properties. Measuring transducers are used both for measure the force excitation by means of piezoelectric dynamometer and accelerometer for measure the response. Both transducers are incorporated in a case of titanium and forming an impedance head. The last is expected to accept the both signals amplified in charge amplifier.

Following the working hypothesis and assuming that the internal damping of the long-rod material is of the "Solid Type I" [1] and that E_ω and the energy loss factor $\delta_{E\omega}$ are independent of frequency and both can be treated as constant quantities E and δ_E . For the material of the samples enrolled in the experiment protocol the expression for P and q [1] are simplified to:

$$P = \frac{nl}{D_E} \left(\frac{D_E + 1}{2} \right)^{0.5} \quad (11)$$

and

$$q = -\frac{nl}{D_E} \left(\frac{D_E - 1}{2} \right)^{0.5}, \quad (12)$$

where

$$D_E = \sqrt{1 + \delta_E^2} \quad (13)$$

For the normalized driving-point impedance Z_0 [1], [4], [8] taking into account the long-rod lumped mass M_R and division by $j\omega$ it can be obtain measure of apparent (or dynamic) mass, $Am(\omega)$:

$$Am(\omega)_{DP} = \frac{Z_0}{j\omega M_R} = -\frac{\cot(P + jq)}{(P + jq)} \quad (14)$$

$$= \frac{(-\cos P \cdot \cosh q + j \sin P \cdot \sinh q)}{(P + jq)(\sin P \cdot \cosh q + j \cos P \cdot \sin q)}$$

The magnitude of the driving-point apparent mass is:

$$|Am(\omega)|_{DP} = \left| \frac{Z_0}{j\omega M_R} \right| = \quad (15)$$

$$\sqrt{\frac{\cos^2 P + \sinh^2 q}{(P^2 + q^2)(\sin^2 P + \sinh^2 q)}}$$

By introducing Eq. (9) and Eq. (10), the phase $\varphi(\omega)$ is presented as:

$$\varphi(\omega)_{DP} = \tan^{-1} \left[\frac{\left(\frac{\sin P \cdot \cos P}{\sinh q \cdot \cosh q} - \frac{q}{P} \right)}{\left(1 + \frac{\sin P \cdot \cos P}{\sinh q \cdot \cosh q} \cdot \frac{q}{P} \right)} \right] \quad (16)$$

To study the wave reflection of the longitudinal vibration it is useful to measure transfer impedance or transfer apparent mass. For the magnitude of transfer apparent mass it can be written:

$$|Am(\omega)|_{TR} = \sqrt{\frac{\sin^2 P + \sinh^2 q}{P^2 + q^2}}, \quad (17)$$

and for the phase angle:

$$\varphi(\omega)_{TR} = \quad (18)$$

$$\tan^{-1} \frac{q \cdot \sinh q \cdot \cos P - P \cdot \sin P \cdot \cosh q}{P \cdot \sinh q \cdot \cos P + q \cdot \sin P \cdot \cosh q}$$

The results from driving-point apparent mass $Am(f)$ (magnitude) are presented in Fig. 5. As it was expected, the driving-point was showing relatively good coherence with the theoretical line of the model (damping coefficient $\zeta = 0,12$). At low frequency from 2 Hz to 6 Hz, diverts

from the theory line, due to the inherent piezo-electric effect.

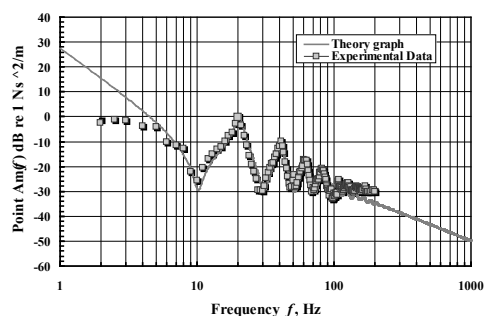


Fig. 5. Driving-point apparent mass $A_m(f)$ (magnitude) of long-rod excited longitudinally by random noise (Gauß)

Driving point apparent mass (phase) (Fig. 6) was used to generate the theoretical phase line (damping coefficient $\zeta = 0,12$) plus two supported lines with damping coefficients $\zeta = 0,01$ and $\zeta = 1$, respectively.

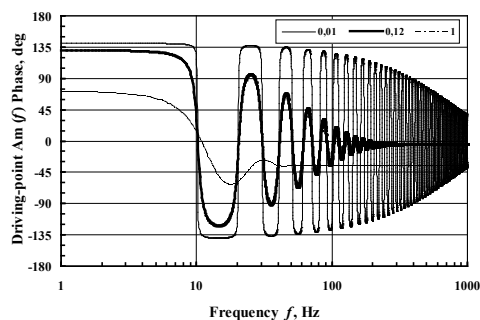


Fig. 6. Driving-point apparent mass $A_m(f)$ (phase) of long-rod excited longitudinally by random noise (Gauß)

3. Hammer Excitation

Another easy, practical and popular method of excitation is through use of an impactor or hammer. Although this type of test places greater demands on the analysis phase of the measurement process, it is a relatively simple means of excitation the structure into vibration.

The waveform produced by an impact is a transient (short duration) energy transfer event. The spectrum is continuous with a maximum amplitude at 0 Hz and decaying amplitude with increasing frequency.

The spectrum has a periodic structure with zero forces at frequency n/T intervals, where n is an integer and T - the effective duration of the transient. The useful frequency range is

from 0 Hz to a frequency F , at which point the spectrum magnitude has decayed by 10 to 20 dB

The duration and thus, the shape of the spectrum, of an impact is determined by the mass and the stiffness of both, the impactor and the structure. The stiffness of the hammer tip determines the spectrum. The hammer tip acts as a mechanical filter. From the theoretical point of view this analogy is not strictly correct since the tip does not filter the energy. It determines the frequency range, where the available energy is concentrated. Selection of the tip stiffness enables the cut-off frequency to be chosen.



Fig. 7. Impact excitation by means of modal testing hammer

Force-sensitive piezo-electrical transducer (Fig. 7) was designed for the purpose of a modal hammer supplement.

Main characteristics of the force transducer are: charge sensitivity $S_q = 0.113$ pC/N, maximum force $F_{\max} = 15350$ N; resonance frequency $f_R = 42$ kHz; transducer mass $m = 0.920$ kg, connector BNC. Total mass: 3,160 kg; 0,1 - 31,6 mV/Unit.

The impact with rubber tip, duration of the impact $D = 1,2$ ms and amplitude 0,823 N was performed. The frequency range was obtained by *Fourier G(f)*, at cut-off frequency $f = 487,5$ Hz (-3 dB) was shown in Fig. 8.

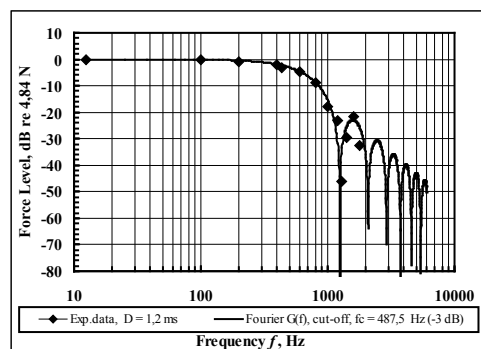


Fig. 8. Force level, dB re 4,84N by impact of 0,823 N

4. Discussion and conclusion

The simple examples show some solutions obtained from nature objects.

In any case of mobility measurement one can expect both random noise in the input and random noise in the output.

Impact excitation and pseudo-random excitation are assessed better by the estimator $H_1(\omega)$. It is used also for the anti-resonances.

When the impact is applied at the nod points represents acoustic noise.

Transients with short duration are treated as infinite period by the use of FFT. For free vibration decay the choice of the exponential window is the simplest solution.

It becomes clear that the structures with high resonances and deep anti-resonances can not be assessed by only one estimator. For anti-resonances $H_1(\omega)$, is suitable, since the noise at the output is more important.

Resonances and random excitation will be better assessed by the estimator $H_2(\omega)$. It cancels noise at the input and it is less sensitive to leakage.

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REFERENCE

- [1] J. C. Snowdon. Vibration and shock in damped mechanical systems. John Wiley & Sons, Inc. 1968.
- [2] Braccesi, C, C., M. Carfagni. Using Experimental Modal Analysis to Simulate Structural Dynamic Modifications. Technical Review Brüel & Kjær Publication. Nærum. Denmark. 1988 pp1-22.
- [3] Broch, J.T., Mechanical Vibration and Shock Measurements, Brüel & Kjær Naerum, Denmark. 1984.
- [4] Cremer, L.,Hechl, M, Körperschall Physikalische Grundlagen und technishe Anwendungen, Springer Verlag, ISBN 3-540-54631-6, 1996. pp 95..180.
- [5] Doebelin, E. O. Measurement Systems. McGraw Hill Incl. Auckland. London. 1983.
- [6] Ewins, D. J. Modal Testing: Theory and Practice, Res. Studies Press Ltd. Letchwoth. England. 1986.
- [7] Heckl, M., Müller. Taschenbuch der Technischen Akustik, Springer-Verlag Berlin Heidelberg New York 1995, pp 248-260.
- [8] Randal, R. Frequency Analysis. Brüel&Kjaer Naerum, Denmark. 1987.